Perpendicular Bisectors and Angle Bisectors of a Triangle:

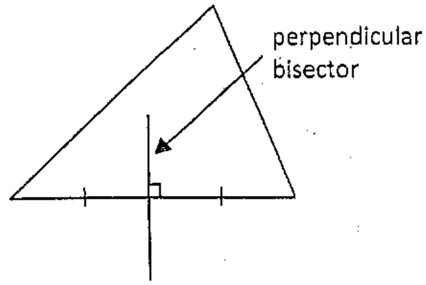
There are four special lines associated with triangles:

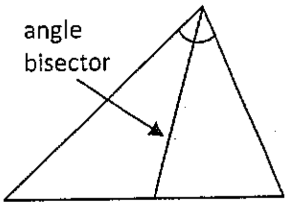
perpendicular bisectors, angle bisectors, altitudes and medians

In this section you are going to learn about perpendicular bisectors and angle bisectors.

Perpendicular bisector:

A perpendicular bisector of a line segment is a line that is perpendicular to the line segment at its midpoint.

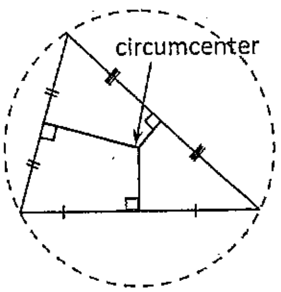




Angle Bisector:

An angle bisector is a line that divides an angle into two congruent angles.

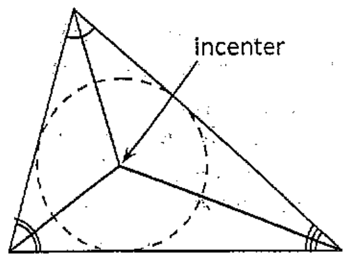
As with altitudes and medians, there are two theorems in geometry that tell us that for any triangle:



- The perpendicular bisectors are concurrent.

The intersection point of the perpendicular bisectors is called the **circumcenter.**

The circumcenter is equidistant for the vertices of the triangle. This means that the circumcenter is the center of a circle that can be drawn around the triangle so that all vertices lie on the circle. We say that such a circle **circumscribes** the triangle.



- The angle bisectors are concurrent.

The intersection point of the altitudes is called the **incenter.**

The incenter is equidistant from the sides of the triangle.

This means that the incenter is the center of a circle that can be **inscribed** inside the triangle so that all sides of the triangle are tangent to the circle.

Example 1: Find the center of the circle that circumscribes triangle OPS.

Solution:

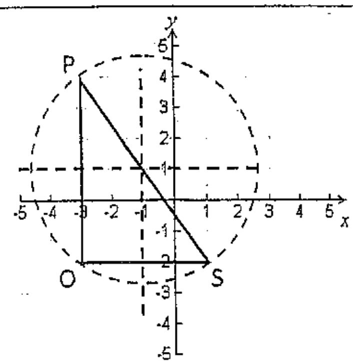
Two of the perpendicular bisectors of the sides of the triangle are:

the line $y = 1$ and the line $x = -1$.

These lines intersect at point (-1, 1).

This point is the center of the circle.

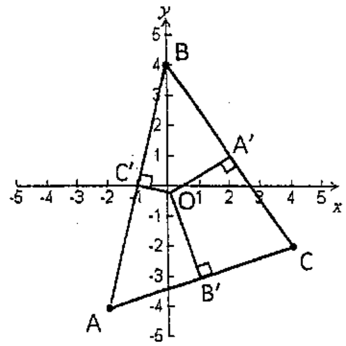
Notice that it was not necessary to find the third perpendicular bisector. Since the three perpendicular bisectors are concurrent, the third bisector will pass through point (-1, 1) also.



Example 2: Consider the triangle with vertices at point: *A* (-2, -4); *B* (0, 4) and *C* (4, -2).
Find the circumcenter of the triangle.

Solution:

Plot the points and draw the triangle and draw the perpendicular bisectors of the triangle.
The circumcenter is at the intersection of the perpendicular bisectors.
We find the equations of the perpendicular bisectors and find where they intersect.
Since we know that the three perpendicular bisectors are concurrent we need only look at the intersection of two of the perpendicular bisectors.



Equation of OA':

The slope of OA' is the negative reciprocal of line BC.

Slope of BC is $\frac{4-(-2)}{0-4}=\frac{6}{-4}=-\frac{3}{2}$, so slope of OA' = $\frac{2}{3}$

Point A' is the midpoint of line BC: A' = $\left(\frac{0+4}{2},\frac{4-2}{2}\right)=(2,1)$

Equation of OA': $y=\frac{2}{3}x+b$

Find *b* by plugging in point A' (2, 1): $1=\frac{2}{3}(2)+b\Rightarrow b=1-\frac{4}{3}=-\frac{1}{3}$

Equation of OA': $y=\frac{2}{3}x-\frac{1}{3}$

Equation of OB':

The slope of OB' is the negative reciprocal of line AC.

Slope of AC: $\frac{-2-(-4)}{4-(-2)}=\frac{2}{6}=\frac{1}{3}$, so slope of OB' = -3

Point B' is the midpoint of line AC: B' = $\left(\frac{-2+4}{2},\frac{-4-2}{2}\right)=(1,-3)$

Equation of OB': $y=-3x+b$

Find *b* by plugging in point B'(1, -3): $-3=-3(1)+b\Rightarrow b=0$

Equation of OA': $y=-3x$

Circumcenter:

The circumcenter is at the intersection of the lines OA' and OB'.

Set the equations equal to each other: $-3x=\frac{2}{3}x-\frac{1}{3}$

Multiply all terms by 3: $-9x=2x-1\Rightarrow 1=11x\Rightarrow x=\frac{1}{11}$ or x = 0.09

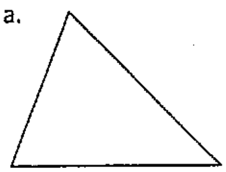
Find y by plugging in the value of x into one of the equations: $y=-3\left(\frac{1}{11}\right)\Rightarrow y=-\frac{3}{11}$ or y = -0.27

Therefore, the circumcenter is at point (0.09, -0.27)

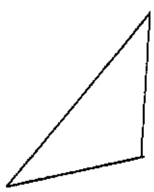
Practice Problems:

1. Construct the angle bisector of each triangle. Then use the incenter to construct an inscribed circle.

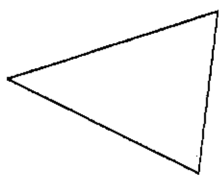
a.



b.



c.



3. A triangle has vertices at A(0, 3); B(5 , 1) and C(1 , -4). Find the circumcenter of the triangle.

4. A triangle has vertices at A(-4, 4); B(-5 , -3) and C(3, 0). Find the circumcenter of the triangle.

5. The points of concurrency for the lines and segments listed in a) – d) have been drawn on the triangle below. Match the points with the lines and segments.

a. perpendicular bisectors
b. angle bisectors
c. medians
d. altitudes