Similar Figures:

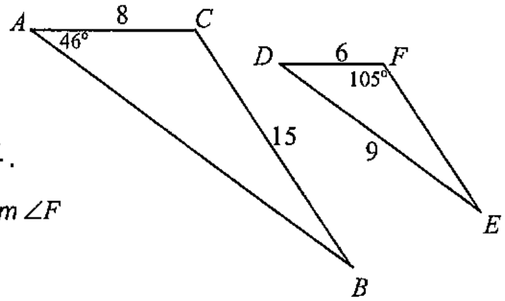
Two figures that have the same shape but not necessarily the same size are **similar** (~) Two polygons are similar if:

- Corresponding angles are congruent and,
- Corresponding sides are proportional.

The ratio of lengths of corresponding sides is called the **similarity ratio.** You can write the ratio as *a* to *b*, *a* : *b* or as $\frac{a}{b}$.

Example 1 : $\Delta ABC \sim \Delta DEF$. Find:

a. the similarity ratio
b. $m\angle C$
c. *FE*



Solution:

a. Since *AC* and *DF* are corresponding sides, the similarity ratio is $\frac{AC}{DF} = \frac{8}{6} = \frac{4}{3}$.

b. $\angle C$ corresponds to $\angle F$, so $m\angle C = m\angle F$ so $m\angle C = 105°$.

c. Write a proportion to solve for *BC*:

$$\frac{BC}{FE} = \frac{AC}{DF} \Rightarrow \frac{15}{FE} = \frac{4}{3}$$

$$FE = \frac{15 \cdot 3}{4} \Rightarrow FE = 11.25$$

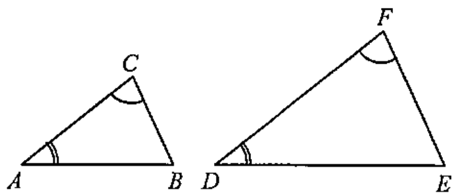
Proving Triangles Similar:

There are several ways to show that two triangles are similar.

1. Angle- Angle Similarity (AA):

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

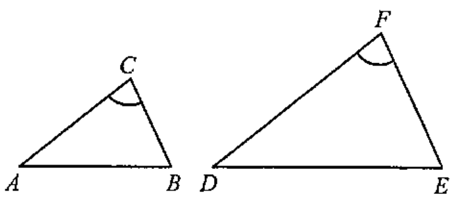
$\Delta ABC \sim \Delta DEF$



2. Side-Angle-Side Similarity (SAS)

If an angle of one triangle is congruent to an angle of a second triangle and the sides including the two angles are proportional, then the triangles are similar.

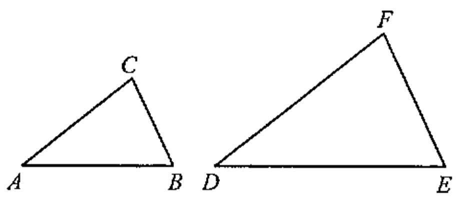
If $\angle C = \angle F$ and $\frac{AC}{DF} = \frac{BC}{EF}$ then $\Delta ABC \sim \Delta DEF$



3. Side-Side-Side Similarity (SSS)

If the corresponding sides of two triangles are proportional, then the triangles are similar.

If $\frac{AC}{DF} = \frac{BC}{EF} = \frac{AB}{DE}$ then $\Delta ABC \sim \Delta DEF$



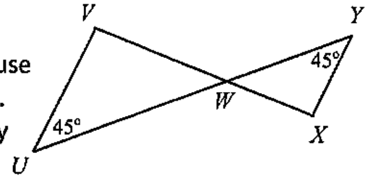
Example 2: Explain why the triangles are similar. Write a similarity statement.

Solution:

$\angle VWU \cong \angle YWX$ because they are vertical angles.

$\angle U \cong \angle Y$ because they are both 45°.

$\Delta WVU \sim \Delta WXY$ by AA Similarity.



Example 3: Show that the triangles are similar and find the length *AC*.

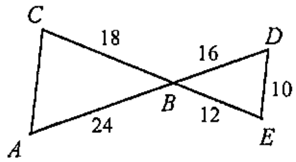
Solution:

$\angle ABC \cong \angle EBD$ because they are vertical angles.

$\frac{AB}{BD} = \frac{24}{16} = \frac{3}{2}$ and $\frac{BC}{BE} = \frac{18}{12} = \frac{3}{2}$

$\Delta ABC \sim \Delta EBD$ by ASA Similarity.

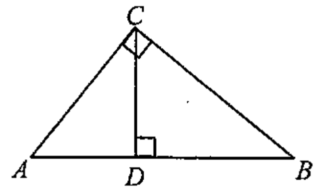
$\frac{AC}{DE} = \frac{3}{2} \Rightarrow AC = \frac{3 \cdot 10}{2} \Rightarrow AC = 15$



4. Similarity in Right Triangles:

The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and to each other.

$\Delta ABC \sim \Delta ACD \sim \Delta CBD$



Example 4: Solve for *x* and *y*.

Solution:

Draw the three right triangles so that they are the same orientation.

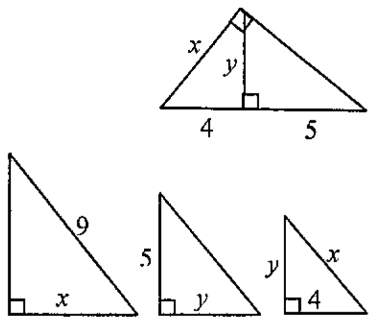
The three triangles are similar to each other.

So, using the large and small triangles:

$\frac{9}{x} = \frac{x}{4}$ or $x^2 = 36 \Rightarrow x = 6$

Using the medium and small triangles:

$\frac{5}{y} = \frac{y}{4}$ or $y^2 = 20 \Rightarrow y = 2\sqrt{5}$



5. Side-Splitter Theorem:

If a line is parallel to one side of a triangle and intersect the other two sides then it divides those sides proportionally.

Example 5: Solve for *x*.

Solution:

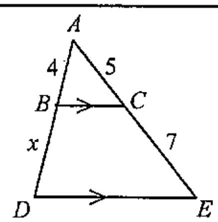
BC ∥ *DE*, therefore $\Delta ABC \sim \Delta ADE$

$\frac{AB}{AD} = \frac{AC}{AE}$ or $\frac{4}{4+x} = \frac{5}{12}$

So: $5(4+x) = 48$

$20 + 5x = 48 \Rightarrow 5x = 28$ or $x = 5.6$

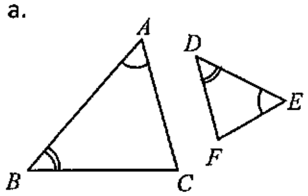
We can also say: $\frac{AB}{BD} = \frac{AC}{CE}$ or $\frac{4}{x} = \frac{5}{7}$. So: $x = \frac{28}{5}$ or $x = 5.6$



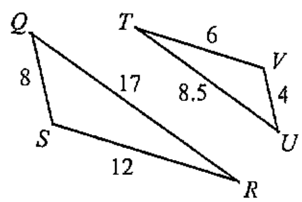
Practice Problems:

1. Are the triangles similar? If so, name the theorem that proves the similarity.

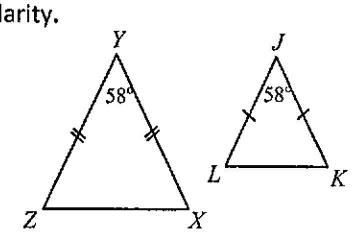
a.



b.

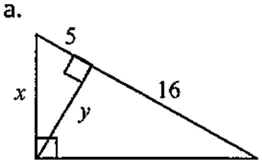


c.

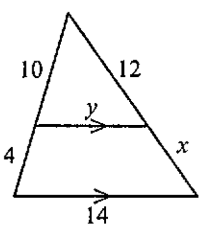


2. Find the values of *x* and *y* in the following triangles.

a.



b.



c.